Answers to problems

2. Characteristics of fluids

1. kg m/s^2
2. Viscosity: Pa s, Kinematic viscosity: m^2/s
3. $v = 0.001\,m^3/kg$
4. 2.06×10^7 Pa
5. $h = \dfrac{2T\cos\theta}{\rho g b}$, $h = 1.48$ cm
6. 291 Pa
7. 9.15×10^{-3} N
8. 1.38 N
9. 1461 m/s

3. Fluid statics

1. 6.57×10^7 Pa
2. (a) $p = p_0 + \rho g H$,
   (b) $p = p_0 - \rho g H$,
   (c) $p = p_0 + \rho' g H' - \rho g H$
3. (a) $p_1 - p_2 = (\rho' - \rho) g H + \rho g H_1$,
   (b) $p_1 - p_2 = (\rho - \rho') g H$
4. 50 mm
5. Total pressure $P = 9.56 \times 10^5$ N, $h_c = 6.62$ m

6. 2.94×10^4 N, 5.87×10^4
7. 9.84×10^3 N
8. Force acting on the unit width: 1.28×10^6 N, Action point located along the wall from the water surface: 11.6 m
9. 7700 N m
10. Horizontal component $P_x = 1.65 \times 10^5$ N, Vertical component $P_y = 1.35 \times 10^5$ N, total pressure $P = 2.13 \times 10^5$ N, acting in the direction of 39.3° from a horizontal line
11. $976\,\mathrm{m}^3$
12. $h = 0.22\,\mathrm{m}$, $T = 0.55\,\mathrm{s}$
13. $\omega = \frac{1}{r_0}\sqrt{2gh'}$ rad/s, $\omega = 14$ rad/s at $h' = 10$ cm, speed of rotation when the cylinder bottom begins to appear $n = 4.23\,\mathrm{s}^{-1} = 254$ rpm

4. Fundamentals of flow

1. (a) A flow which does not change as time elapses is called a **steady** flow. **Velocity**, **pressure** and **density** of flow in a steady flow are functions of position only, and most of the flows studied in hydrodynamics are steady flows. A flow which changes as time elapses is called an **unsteady** flow. **Velocity**, **pressure** and **density** of flow in an unsteady flow are functions of **time** and **position**. Flows such as when a valve is **opened**/**closed** or the **discharge** from a tank belong to this flow.

   (b) The flow velocity is **proportional** to the radius for a free vortex flow, and is **inversely** **proportional** to the radius for a forced vortex flow.

2. $\Gamma = 0.493\,\mathrm{m}^2/\mathrm{s}$
3. $Re = 6 \times 10^4$, turbulent flow
4. $\frac{dx}{x} = -\frac{dy}{y}$ namely $xy = \text{const}$
5. (a) Rotational flow
   (b) Irrotational flow
   (c) Irrotational flow

6. Water $v_c = 23.3\,\text{cm/s}$, air $v_c = 3.5\,\text{m/s}$
7. $\Gamma = 82\,\text{m}^2/\text{s}$

5. One-dimensional flow

1. See text.
2. $v_1 = 6.79\,\text{m/s}$, $v_2 = 4.02\,\text{m/s}$, $v_3 = 1.70\,\text{m/s}$
3. $p_2 = 39.5\,\text{kPa}$, $p_3 = 46.1\,\text{kPa}$
4. p_0: Atmospheric pressure, p: Pressure at the point of arbitrary radius r

$$p_0 - p = \frac{\rho Q^2}{8\pi^2 h^2}\left(\frac{1}{r^2} - \frac{1}{r_2^2}\right)$$

Total pressure (upward direction) $P = \frac{\rho Q^2}{4\pi h^2}\left[\log\frac{r_2}{r_1} - \frac{1}{2}\left(1 - \frac{r_1^2}{r_2^2}\right)\right]$

5. $v_r = 5.75\,\text{m/s}$, $p_r - p_0 = -1.38 \times 10^4\,\text{Pa}$
6. $t = \frac{2A\sqrt{H}}{Ca\sqrt{2g}}$
7. Condition of section shape $H = \left(\frac{\pi v}{Ca\sqrt{2g}}\right)^2 r^4$

   $Q = 12.9\,\text{m}^3/\text{s}$, $d = 1.29\,\text{mm}$
8. $H = 2.53\,\text{m}$
9. $Q_1 = \frac{1+\cos\theta}{2}Q$, $Q_2 = \frac{1-\cos\theta}{2}Q$, $F = \rho Qv\sin\theta$

   $Q_1 = 0.09\,\text{m}^3/\text{s}$, $Q_2 = 0.03\,\text{m}^3/\text{s}$, $F = 2.53 \times 10^4\,\text{N}$
10. $-7.49\,\text{mH}_2\text{O}$
11. $n = 6.89\,\text{s}^{-1} = 413\,\text{rpm}$, torque $8.50 \times 10^{-2}\,\text{N m}$
12. $F = 749\,\text{N}$

6. Flow of viscous fluids

1. See text.
2. $\frac{1}{r}\frac{\partial(rv)}{\partial r} + \frac{\partial u}{\partial x} = 0$, or $\frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial x} = 0$
3. (a) $u = 6v\left[\frac{y}{h} - \left(\frac{y}{h}\right)^2\right]$,

   (b) $v = \frac{1}{1.5}u_{\max}$

(c) $Q = \frac{h^3}{12\mu}\frac{\Delta p}{l}$,

(d) $\Delta p = \frac{12\mu l Q}{h^3}$

4. (a) $u = 2v\left[1 - \left(\frac{r}{r_0}\right)^2\right]$

(b) $v = \frac{1}{2}u_{\max}$

(c) $Q = \frac{\pi d^4}{128u}\frac{\Delta p}{l}$

(d) $\Delta p = \frac{128\mu l Q}{\pi d^4}$

5. (a) $v = 0.82u_{\max}$,

(b) $r = 0.76r_0$

6. $\varepsilon = 4.57 \times 10^{-5}\,\mathrm{m^2/s}$, $l = 2.01\,\mathrm{cm}$

7. $Q = \frac{\pi d h^3}{12\mu}\frac{\Delta p}{l}$

8. $h_2 = 0.72\,\mathrm{mm}$

9. $\mathrm{LT^{-1}}$

10. 8.16 N

7. Flow in pipes

1, 2, 3, 4. See applicable texts.

5. See applicable text. Error of loss head h is 5α (%)

6. $h = 733$ m at diameter 50 mm, $h = 26.4$ m at diameter 100 mm

7. 24.6 kW

8. Pressure loss $\Delta p = 508$ Pa

9. 3.2 cm H_2O

10. $h_s = 6.82$ cm, $\eta = 0.91$

8. Flow in water channel

1. $i = \frac{4.56}{1000}$

2. From Chézy's equation $Q=40.4\,\text{m}^3/\text{s}$, from Manning's equation $Q=40.9\,\text{m}^3/\text{s}$
3. $Q = 19.3\,\text{m}^3/\text{s}$
4. Flow velocity becomes maximum at $\theta = 257.5°$, $h = 2.44\,\text{m}$ and discharge becomes maximum at $\theta = 308°$, $h = 2.85\,\text{m}$
5. Tranquil flow, $E = 1.52\,\text{m}$
6. $h_c = 0.972\,\text{m}$, $3.09\,\text{m/s}$
7. $Q_{\text{max}} = 14.4\,\text{m}^3/\text{s}$
8. 1.18 m
9. See applicable text.

9. Drag and lift

1. Using Stokes equation, terminal velocity $v = \dfrac{d^2 g}{18v}\left(\dfrac{\rho_s}{\rho_w} - 1\right)$ where d is diameter of a spherical sand particle and ρ_w, ρ_s are density of water and sand respectively.
2. $D = 1450\,\text{N}$, Maximum bending moment $M_{\text{max}} = 3620\,\text{N m}$
3. $D = 2.70\,\text{N}$
4. $\delta_{\text{max}} = 3.2\,\text{cm}$ at wind velocity 4 km/h, $\delta_{\text{max}} = 4.1\,\text{cm}$ at wind velocity 120 km/h
5. $T = 722\,\text{N m}$, $L = 4.54 \times 10^4\,\text{N m/s}$

6, 7. See texts.

8. $D_f = 88.9\,\text{N}$, Required power $P = 133\,\text{N m/s}$
9. $L = 3.57\,\text{N}$
10. $D = 134\,\text{N}$

10. Dimensional analysis and law of similarity

1. Consider v, g, H as the physical influencing quantities and perform dimensional analysis. $v = C\sqrt{gH}$
2. $D = C\mu Ud$
3. $a = C\sqrt{\dfrac{K}{\rho}}$
4. $D = \rho L^2 v^2 f\left(\dfrac{v}{\sqrt{Lg}}\right)$

5. $Q = C\dfrac{d^4}{\mu}\dfrac{\Delta p}{l}$

6. $\delta = xf\left(\dfrac{Ux}{\nu}\right)$

7. $C = f\left(\dfrac{d\sqrt{2\rho\Delta p}}{\mu}\right) = f(Re)$

8. (a) 167 m/s

   (b) 33.3 m/s

   (c) 11.1 m/s

9. Towing velocity for the model $v_m = 2.88\,\text{m/s}$

10. $\dfrac{1}{2.36}$

11. Measurement of flow velocity and flow rate

1. $v = 4.44\,\text{m/s}$
2. $v = 28.5\,\text{m/s}$
3. Mass flow rate $m = 0.325\,\text{kg/s}$
4. $C_c = 0.64$, $C_v = 0.95$, $C = 0.61$

5, 6, 7. See applicable texts.

8. $U = 50\,\text{cm/s}$
9. See applicable texts.
10. Error for rectangular weir is 3%, error for triangular is 5%.

12. Flow of ideal fluid

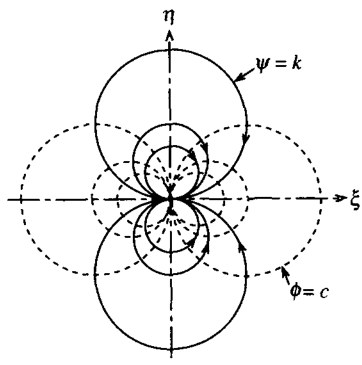
1. $\phi = u_0x + v_0y$, $\psi = u_0y - v_0x$
2. See applicable text.
3. Flow in counterclockwise rotary motion, $v_\theta = \Gamma/2\pi r$, $v_r = 0$, around the origin.
4. $\phi = \dfrac{q}{2\pi}\log r$, $\psi = \dfrac{q}{2\pi}\theta$
5. Putting $r = r_0$, $\psi = 0$, the circumference becomes one stream line. Velocity distribution $v_\theta = -2U\sin\theta$, Pressure distribution $\dfrac{p - p_\infty}{pU^2/2} = 1 - 4\sin^2\theta$

6. The flow around a rectangular corner.

7. Flow in clockwise rotary motion, $v_\theta = -\dfrac{\Gamma}{2\pi r}$, $v_r = 0$, around the origin.

8. $w = Uze^{-ia}$

9.



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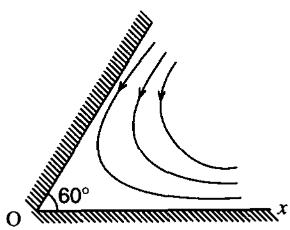

13. Flow of a compressible fluid

1. $\rho = \dfrac{p}{RT} = 1.226\,\mathrm{kg/m^3}$

2. $a = \sqrt{kRT} = 1297\,\mathrm{m/s}$

3. $T_2 = T_1 + \dfrac{1}{2}\dfrac{\kappa - 1}{\kappa}\dfrac{1}{R}(u_1^2 - u_2^2) = 418\,\mathrm{K}$
   $t_2 = 145°\mathrm{C}$
   $p_2 = p_1\left(\dfrac{T_1}{T_2}\right)^{\kappa/(\kappa-1)} = 3.4 \times 10^5\,\mathrm{Pa}$

4. $T_0 = 278.2\,\mathrm{K}$, $t_0 = 5.1°\mathrm{C}$
   $p_0 = 6.81 \times 10^4\,\mathrm{Pa}$
   $\rho_0 = 0.85\,\mathrm{kg/m^3}$

5. $v = 444\,\mathrm{m/s}$

6. $M = 0.73$, $a = \sqrt{\kappa RT} = 325\,\mathrm{m/s}$, $v = aM = 237\,\mathrm{m/s}$

7. $v = 272\,\mathrm{m/s}$

8. $\dfrac{p}{p_0} = 0.45 < 0.528$, $m = 0.0154\,\mathrm{kg/s}$

9. $\dfrac{A_2}{A_*} = 1.66$

10. $A_2 = 2354\,\mathrm{cm}^2$

11. $2.35 \times 10^5\,\mathrm{N}$

12. Mach number 0.58, flow velocity 246 m/s, pressure $2.25 \times 10^5\,\mathrm{Pa}$

14. Unsteady flow

1. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \pm 1.39\,\mathrm{m/s}$, $T = 1.57\,\mathrm{s}$

2. $T = 2\pi\sqrt{\dfrac{l}{g(\sin\theta_1 + \sin\theta_2)}}$

3. $0.69\,\mathrm{m/s}$

4. $t = 1\,\mathrm{min}\ 20\,\mathrm{s}$

5. $a = 837\,\mathrm{m/s}$

6. $\Delta p = 2.51 \times 10^6\,\mathrm{Pa}$

7. $p_{\max} = 1.56 \times 10^6\,\mathrm{Pa}$